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A. Tzinevrakis, D. Tsanakas, E. Mimos

Analytical calculation of the electric field produced by single circuit power lines with horizontal arrangement of the conductors

Power Engineering

Abstract

The electric field produced by electric power lines is usually calculated with the use of numerical methods. However, the analytical calculation of the electric field is preferable, because it leads to a mathematical expression, which shows the field's dependencies from all the line parameters. Initially, a method to derive the analytical formulas of the intensity of the electric field produced by electric power lines, is developed in the present paper. This method is based on the analysis of the intensity of the electric field into the sum of two multipole expansions and in the use of double complex numbers for the simplification of the mathematical expressions. Due to the asymmetry of conductors charges, they are analyzed in their symmetrical components. Afterwards, the accurate analytical formula of the intensity of the electric field for simple circuit power lines with the conductors in horizontal arrangement is given, which is valid at any point in the vicinity of lines. Beyond the accurate formula, simpler approximate formulas are presented, which give the electric field with known precision.

1 Introduction

During the last 25 years the electric and magnetic fields are considered more and more as environmental factors. The International Commission on Non–Ionizing Radiation Protection (ICNIRP) published in 1998 the guidelines [1]. In these guidelines, the limit values are 5kV/m and 100μT for the public exposure and 10kV/m and 500μT for the occupational exposure. The World Health Organization (WHO) and the European Union (E.U.) have adopted the limits of ICNIRP [2,3]. These limits are included in the national legislations of the most countries in the E.U.

In the present paper analytical formulas are developed for the calculation of the intensity of the electric field in the vicinity of single circuit power lines. Corresponding analytical formulas for the calculation of magnetic flux density were published in [4]. The analytical formulas provide the advantage of easy and precise calculation of the field intensities and of a simpler parametrical investigation for evaluating the effect of various parameters, without the use of numerical methods. Therefore, they are appropriate for the assessment of the public exposure, e.g. in the frames of epidemiological studies.

2 Calculation procedure

2.1 Electric field produced by power lines

Fig. 1 shows the model of a simple circuit power line and a point of interest P for the calculation of the intensity of the electric field. It is considered that the line route is straight and that its length is very big in comparison with the distances between the conductors and the distances of conductors from the ground. The three conductors in fig. 1 bring charges $\underline{Q}_a$, $\underline{Q}_b$ and $\underline{Q}_c$, which are sinusoidal varying in time, while their distances from the point P are R_1, R_2 and R_3 respectively. The intensity of electric field $\underline{\mathbf{E}}_k$, which produced by a conductor k in point P, is given by

$$\underline{\mathbf{E}}_k = \frac{\underline{Q}_k}{2\pi\varepsilon_0}\left(\frac{1}{R_k'^2}\mathbf{R}_k' - \frac{1}{R_k^2}\mathbf{R}_k\right) \tag{2.1}$$

where $\varepsilon_0 = \frac{1}{36\pi}10^{-9}\frac{A \cdot s}{V \cdot m}$ is the electric permittivity of free space, $\mathbf{R}_k$ the vector distance from the k conductor to the point P, $\mathbf{R}_k'$ the vector distance from the image of the k conductor to point P and R_k, R_k' the modulus of vectors $\mathbf{R}_k$ and $\mathbf{R}_k'$ respectively.

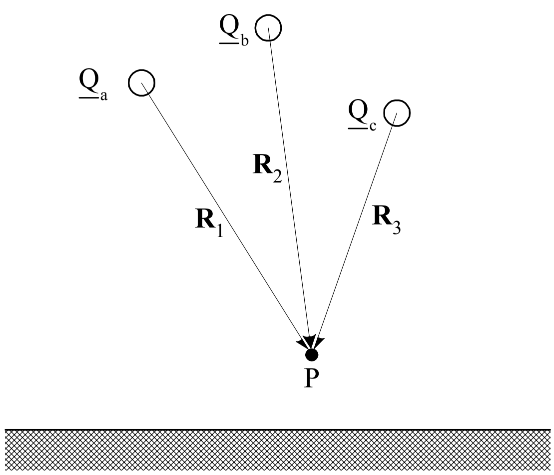


Fig. 1. Single circuit power line model

In the general case, a line with n conductors may be considered. Using the superimposition theorem, the intensity of electric field $\underline{\mathbf{E}}$, produced by the line is equal to the sum of the fields produced by each conductor separately

$$\underline{\mathbf{E}}=\sum_{\kappa=1}^{n}\underline{\mathbf{E}}_k=\frac{1}{2\pi\varepsilon_0}\sum_{\kappa=1}^{n}\underline{Q}_k\left(\frac{1}{R_k^{'2}}\mathbf{R}_k^{'}-\frac{1}{R_k^2}\mathbf{R}_k\right)\Rightarrow$$

$$\underline{\mathbf{E}}=\frac{1}{2\pi\varepsilon_0}\sum_{\kappa=1}^{n}\underline{Q}_k\left(\frac{1}{\overline{\mathbf{R}}_k^{'}}-\frac{1}{\overline{\mathbf{R}}_k}\right) \tag{2.2}$$

Relation (2.2) is usually used for the numerical calculation of the intensity of the electric field. However, the summing operation doesn’t allow this formula to be readily applied for the calculation of the electric field and makes it unsuitable for reaching general conclusions for the electric field properties and its dependencies on the various parameters of the line.

2.2 Double complex numbers and multipole expansions

The relations for the electric field vector are simplified if complex numbers are used for the representation of vector distances. On the other hand, complex numbers are used for the representation of sinusoidal varying quantities in time (phasors), such as conductors charges, so finally double complex numbers are required. Book [5] contains the theoritical basis of double complex numbers, in [4] they are used for the calculation of magnetic flux density and in [6] to express the elliptically polarized vector of the electric field of light.

For the representation of space vectors the set of complex numbers C_1 is used, with the imaginary part j, for example $\mathbf{R}_k=x_k+jy_k$, while for the representation of sinusoidal varying quantities in time, the set C_2 with the imaginary part i is used, for example $\underline{Q}_k=Q_k e^{i\varphi\kappa}$.

Fig 2 shows once again the model of single circuit power line as well as the images of conductors. Each conductor k is characterized by the vector distance $\mathbf{d}_k$ from the reference point O, which is a central point of the real conductors system. The image of each conductor k is characterized by the vector distance $\mathbf{d}_k^{'}$ from the reference point O´, which is also a central point for the images conductors system. The points O and O´ are close to but not necessarily the centers of the conductors systems. Their choice is a matter of experience and is made in order to simplify the mathematical expressions.

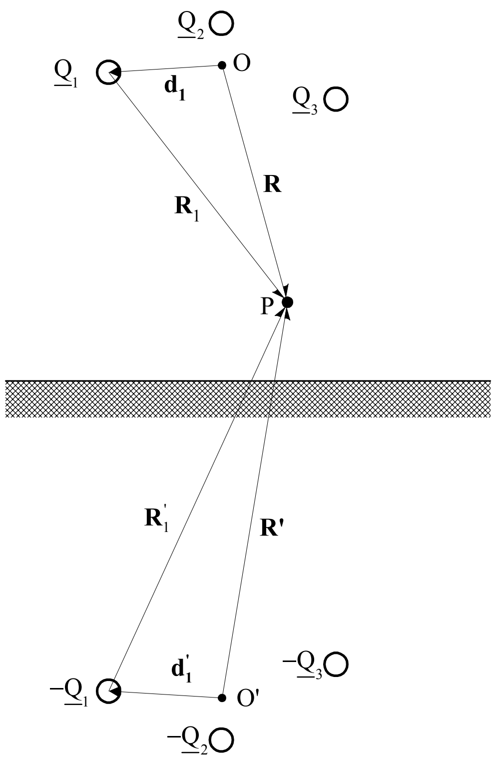


Fig. 2. Model of a single circuit power line with the conductors images

Replacing $\mathbf{R}_k = \mathbf{R} - \mathbf{d}_k$ and $\mathbf{R}_k' = \mathbf{R}' - \mathbf{d}_k'$ in (2.2), it becomes (2.3).

$$\underline{\mathbf{E}} = \frac{1}{2\pi\varepsilon_0} \sum_{k=1}^{n} \underline{Q}_k \left(\frac{1}{\overline{\mathbf{R}}' - \overline{\mathbf{d}}_k'} - \frac{1}{\overline{\mathbf{R}} - \overline{\mathbf{d}}_k} \right) \tag{2.3}$$

Using the well-known relations (2.4) in (2.3), the intensity of the electric field results as the sum of two multipole expansions, one from the real conductors system and the other from the images conductors system, relation (2.5).

$$\left(\overline{\mathbf{R}}' - \overline{\mathbf{d}}_k' \right)^{-1} = \sum_{\lambda=1}^{\infty} \frac{\overline{\mathbf{d}}_k'^{\,\lambda-1}}{\overline{\mathbf{R}}'^{\,\lambda}} \tag{2.4a}$$

$$\left(\overline{\mathbf{R}} - \overline{\mathbf{d}}_k \right)^{-1} = \sum_{\lambda=1}^{\infty} \frac{\overline{\mathbf{d}}_k^{\,\lambda-1}}{\overline{\mathbf{R}}^{\lambda}} \tag{2.4b}$$

$$\underline{\mathbf{E}}=\frac{1}{2\pi\varepsilon_0}\sum_{k=1}^{n}\underline{Q}_k\left(\sum_{\lambda=1}^{\infty}\frac{\overline{\mathbf{d}}_k'^{\,\lambda-1}}{\overline{\mathbf{R}}'^{\lambda}}-\sum_{\lambda=1}^{\infty}\frac{\overline{\mathbf{d}}_k^{\,\lambda-1}}{\overline{\mathbf{R}}^{\lambda}}\right)\Rightarrow$$

$$\underline{\mathbf{E}}=\sum_{\lambda=1}^{\infty}\underline{\mathbf{E}}'_{(\lambda)}+\sum_{\lambda=1}^{\infty}\underline{\mathbf{E}}_{(\lambda)} \tag{2.5}$$

The terms $\underline{\mathbf{E}}'_{\lambda}$ and $\underline{\mathbf{E}}_{\lambda}$ are called λ order terms and are given by the relations (2.6)

$$\underline{\mathbf{E}}'_{(\lambda)}=\frac{1}{2\pi\varepsilon_0}\cdot\frac{\underline{\underline{\mathbf{M}}}'_{\lambda}}{\overline{\mathbf{R}}'^{\lambda}} \tag{2.6a}$$

$$\underline{\mathbf{E}}_{(\lambda)}=-\frac{1}{2\pi\varepsilon_0}\cdot\frac{\underline{\underline{\mathbf{M}}}_{\lambda}}{\overline{\mathbf{R}}^{\lambda}} \tag{2.6b}$$

where the factors $\underline{\mathbf{M}}'_{\lambda}$ and $\underline{\mathbf{M}}_{\lambda}$ are called λ order moments for the images conductors system and the real conductors system respectively and are given by the relations (2.7).

$$\underline{\mathbf{M}}'_{\lambda}=\sum_{\kappa=1}^{n}\underline{Q}_k\overline{\mathbf{d}}_k'^{\,\lambda-1} \tag{2.7a}$$

$$\underline{\mathbf{M}}_{\lambda}=\sum_{\kappa=1}^{n}\underline{Q}_k\overline{\mathbf{d}}_k^{\,\lambda-1} \tag{2.7b}$$

The multipole expansion, which appears in relation (2.5), is a series of infinite terms, each inversely proportional to an increasing power of the distance and has been used in [4] and [7] for the calculation of magnetic field produced by electric power lines.

2.3 Analysis in symmetrical components

The conductors charges $\underline{Q}_a$, $\underline{Q}_b$ and $\underline{Q}_c$ constitute an asymmetrical system, which impedes the calculation of the intensity of the electric field. The asymmetrical system of charges is analyzed in three symmetrical three phase systems, the positive, the negative and the zero sequence system, according to relation (2.8).

$$\begin{bmatrix} \underline{Q}_0 \\ \underline{Q}_1 \\ \underline{Q}_2 \end{bmatrix} = \frac{1}{3} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \cdot \begin{bmatrix} \underline{Q}_a \\ \underline{Q}_b \\ \underline{Q}_c \end{bmatrix} \tag{2.8}$$

where $\underline{Q}_0$, $\underline{Q}_1$, $\underline{Q}_2$ the positive, negative and zero sequence charges respectively and $\underline{a} = e^{i2\pi/3}$.

The intensity of the electric field is calculated for each one of the components of charges separately and the final accurate formulas arise from the superimposition of the results.

3 Electric field of single circuit power lines with conductors in horizontal arrangement

3.1 Accurate formula of the electric field

Fig.3 shows a single circuit power line with the conductors in horizontal arrangement.

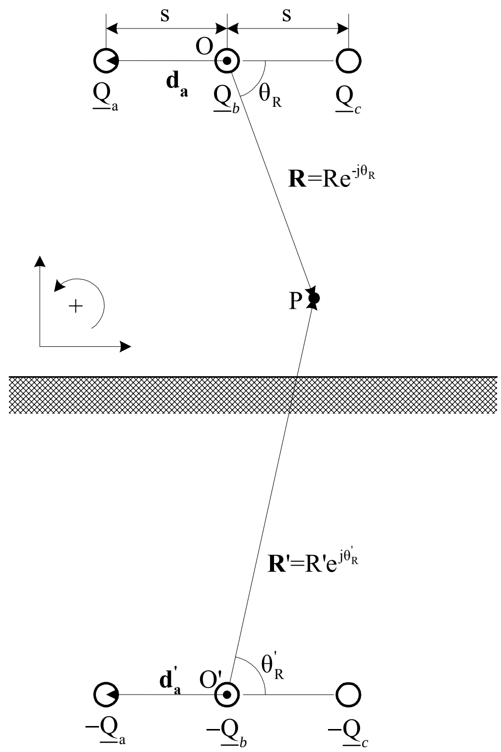


Fig. 3. Model of a single circuit power line with the conductors in horizontal arrangement

Considering the middle conductor as the reference point for both the real conductors system and the images conductors system, as shown in fig.3, the distances of the conductors from these points are given in (3.1).

$$\mathbf{d}_a=-s \quad , \quad \mathbf{d}_b=0 \quad , \quad \mathbf{d}_c=s \tag{3.1a}$$

$$\mathbf{d}_a^{'}=-s \quad , \quad \mathbf{d}_b^{'}=0 \quad , \quad \mathbf{d}_c^{'}=s \tag{3.1b}$$

where s is the distance between two adjacent conductors, shown in fig. 3.

Due to $\mathbf{d_k}=\mathbf{d}_k^{'}$ for all the conductors k, the moments $\underline{\mathbf{M}}_\lambda$ and $\underline{\mathbf{M}}_\lambda^{'}$, which are given in relations (2.7), are equal,

$$\underline{\mathbf{M}}_\lambda=\underline{\mathbf{M}}_\lambda^{'} \tag{3.2}$$

The positive, negative and zero sequence charges are calculated according to relation (2.8) and for all the distances s between the conductors, arises that the charges angles θ_1, θ_2 and θ_0 remain invariable and equal with:

$$\theta_1=\frac{2\pi}{3} \quad , \quad \theta_2=-\frac{2\pi}{3} \quad , \quad \theta_0=\pi \tag{3.3}$$

Considering that the conductors bring only the positive sequence charges, then each conductor's charge is given in relation (3.4)

$$\underline{Q}_a=\underline{Q}_1 \quad , \quad \underline{Q}_b=\underline{a}^2\underline{Q}_1 \quad , \quad \underline{Q}_c=\underline{a}\underline{Q}_1 \tag{3.4}$$

where according to relation (3.3), results

$$\underline{Q}_1=Q_1\cdot e^{i\frac{2\pi}{3}} \tag{3.5}$$

Replacing relations (3.1), (3.4) and (3.5) in relations (2.7), (2.6) and (2.5) and taking into consideration (3.2), results the intensity of the electric field $\underline{\mathbf{E}}_1$ that comes from the positive sequence charge and is given in (3.6).

$$\underline{\mathbf{E}}_1=\frac{-iQ_1 s}{2\pi\varepsilon_0\overline{\mathbf{R}}\overline{\mathbf{R}'}}\cdot\frac{\left(\overline{\mathbf{R}}-\overline{\mathbf{R}'}\right)}{\left(\overline{\mathbf{R}'}^2-s^2\right)\left(\overline{\mathbf{R}}^2-s^2\right)}\cdot\left[\sqrt{3}\overline{\mathbf{R}}\overline{\mathbf{R}'}\left(\overline{\mathbf{R}}+\overline{\mathbf{R}'}\right)-is\left(\overline{\mathbf{R}}^2+\overline{\mathbf{R}}\overline{\mathbf{R}'}+\overline{\mathbf{R}'}^2-s^2\right)\right] \tag{3.6}$$

Considering that the conductors bring only the negative sequence charges, then each conductor's charge is given in relation (3.7)

$$\underline{Q}_a=\underline{Q}_2 \quad , \quad \underline{Q}_b=\underline{a}\underline{Q}_2 \quad , \quad \underline{Q}_c=\underline{a}^2\underline{Q}_2 \tag{3.7}$$

where according to relation (3.3)

$$\underline{Q}_2 = Q_2 \cdot e^{-i\frac{2\pi}{3}} \tag{3.8}$$

Replacing relations (3.1), (3.7) and (3.8) in relations (2.7), (2.6) and (2.5) and taking into consideration (3.2), results the intensity of the electric field $\underline{\mathbf{E}}_2$ that comes from the negative sequence charge and is given in (3.9).

$$\underline{\mathbf{E}}_2 = \frac{iQ_2 s}{2\pi\varepsilon_0 \overline{\mathbf{R}}\overline{\mathbf{R}'}} \cdot \frac{\left(\overline{\mathbf{R}} - \overline{\mathbf{R}'}\right)}{\left(\overline{\mathbf{R}'}^2 - s^2\right)\left(\overline{\mathbf{R}}^2 - s^2\right)} \cdot \left[\sqrt{3}\overline{\mathbf{R}}\overline{\mathbf{R}'}\left(\overline{\mathbf{R}} + \overline{\mathbf{R}'}\right) + is\left(\overline{\mathbf{R}}^2 + \overline{\mathbf{R}}\overline{\mathbf{R}'} + \overline{\mathbf{R}'}^2 - s^2\right)\right] \tag{3.9}$$

Considering that the conductors bring only the zero sequence charges, then each conductor’s charge is given in relation (3.10)

$$\underline{Q}_a = \underline{Q}_0 \quad , \quad \underline{Q}_b = \underline{Q}_0 \quad , \quad \underline{Q}_c = \underline{Q}_0 \tag{3.10}$$

where according to relation (3.3)

$$\underline{Q}_0 = Q_0 \cdot e^{i\pi} \tag{3.11}$$

Replacing relations (3.1), (3.10) and (3.11) in relations (2.7), (2.6) and (2.5) and taking into consideration (3.2), results the intensity of the electric field $\underline{\mathbf{E}}_0$ that comes from the zero sequence charge and is given in (3.12).

$$\underline{\mathbf{E}}_0 = \frac{-Q_0 s}{2\pi\varepsilon_0 \overline{\mathbf{R}}\overline{\mathbf{R}'}} \cdot \frac{\left(\overline{\mathbf{R}} - \overline{\mathbf{R}'}\right)}{\left(\overline{\mathbf{R}'}^2 - s^2\right)\left(\overline{\mathbf{R}}^2 - s^2\right)} \cdot \left(\frac{3}{s}\overline{\mathbf{R}}^2\overline{\mathbf{R}'}^2 - s\overline{\mathbf{R}}^2 - s\overline{\mathbf{R}'}^2 + 2s\overline{\mathbf{R}}\overline{\mathbf{R}'} + s^3\right) \tag{3.12}$$

The intensity of the electric field $\underline{\mathbf{E}}$, at any point in the vicinity of the single circuit line, is given by relation (3.13), from which results its final expression (3.14).

$$\underline{\mathbf{E}} = \underline{\mathbf{E}}_1 + \underline{\mathbf{E}}_2 + \underline{\mathbf{E}}_0 \tag{3.13}$$

$$\underline{\mathbf{E}} = \frac{s}{2\pi\varepsilon_0 \overline{\mathbf{R}}\overline{\mathbf{R}'}} \cdot \frac{\left(\overline{\mathbf{R}} - \overline{\mathbf{R}'}\right)}{\left(\overline{\mathbf{R}'}^2 - s^2\right)\left(\overline{\mathbf{R}}^2 - s^2\right)} \cdot \left[As\left(\overline{\mathbf{R}}^2 + \overline{\mathbf{R}'}^2 - s^2\right) + Bs\overline{\mathbf{R}}\overline{\mathbf{R}'} + \frac{C}{s}\overline{\mathbf{R}}^2\overline{\mathbf{R}'}^2 + iD\overline{\mathbf{R}}\overline{\mathbf{R}'}\left(\overline{\mathbf{R}} + \overline{\mathbf{R}'}\right)\right] \tag{3.14}$$

Factors A, B, C and D have dimensions of charges and are given in relation (3.15)

$$A = -Q_1-Q_2+Q_0, \quad B = -Q_1-Q_2-2Q_0, \quad C = -3Q_0, \quad D = -\sqrt{3}Q_1 + \sqrt{3}Q_2 \qquad (3.15)$$

Relation (3.14) shows the intensity of the electric field as a double complex number. The rms value of the electric field E is equal to the modulus of the double complex number $\underline{\mathbf{E}}$ and is given in relation (3.16).

$$E=\frac{s}{2\pi\varepsilon_0 RR'}\cdot\left[\frac{\left(R^2+R'^2-2RR'\cos(\theta_R+\theta_{R'})\right)}{\left(R'^4+s^4-2R'^2s^2\cos2\theta_{R'}\right)\left(R^4+s^4-2R^2s^2\cos2\theta_R\right)}\right]^{1/2}\cdot$$
$$[\left(AsR^2\cos2\theta_R+AsR'^2\cos2\theta_{R'}-As^3+BsRR'\cos(\theta_R-\theta_{R'})+\frac{C}{s}R^2R'^2\cos(2\theta_R-2\theta_{R'})\right)^2+$$
$$+\left(AsR^2\sin2\theta_R-AsR'^2\sin2\theta_{R'}+BsRR'\sin(\theta_R-\theta_{R'})+\frac{C}{s}R^2R'^2\sin(2\theta_R-2\theta_{R'})\right)^2+$$
$$+\left(DR^2R'\cos(2\theta_R-\theta_{R'})+DRR'^2\cos(\theta_R-2\theta_{R'})\right)^2+$$
$$+\left(DR^2R'\sin(2\theta_R-\theta_{R'})+DRR'^2\sin(\theta_R-2\theta_{R'})\right)^2]^{1/2} \qquad (3.16)$$

Factors A, B, C and D are given in (3.15), while the angles θ_R και $\theta_{R'}$ are shown in fig. 3.

3.2 Approximate formulas of the electric field

Relation (3.16) is the accurate formula of the intensity of the electric field and depends on the positive, negative and zero sequence charges. This formula is very extensive. A simpler approximate formula may be derived taking into account only one of the symmetrical components of the charges.

Fig. 4 shows the effect from each one of the symmetrical components of the charges in the intensity of the electric field, for a typical 150kV power line with the conductors in horizontal arrangement. From fig. 4 results that the positive sequence charge has the bigger effect in the electric field not only close to but also far from the line. However, choosing only the positive sequence charge for the calculation of the electric field, the deviation from the accurate values, mainly close to the line, is considerable.

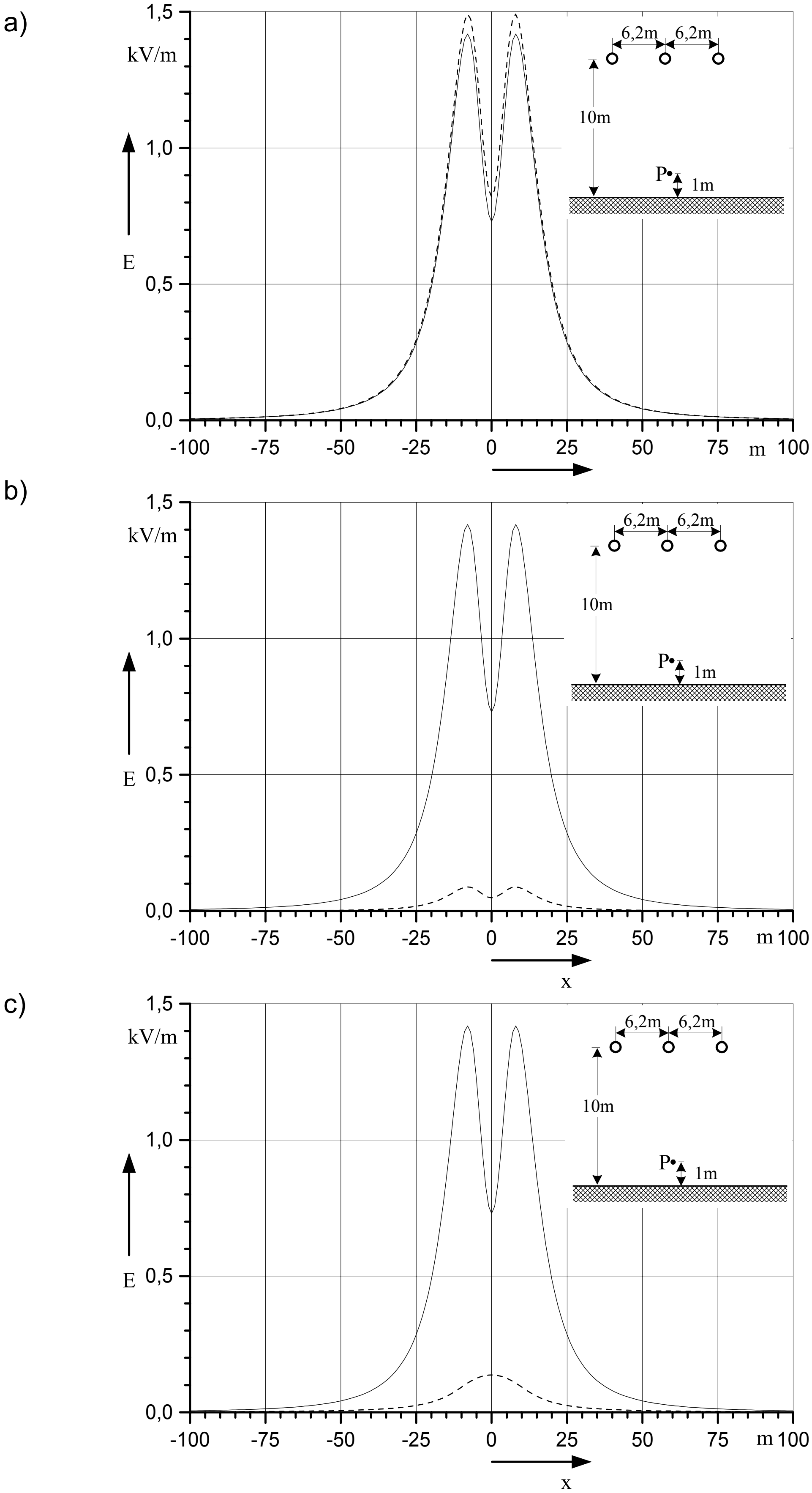


Fig. 4. a) Positive, b) negative and c) zero sequence charge effect in the electric field E that produced by a typical 150kV single circuit line with horizontal arrangement of conductors
——— Accurate values by formula (3.16) ---------- Symmetrical components effect

Considering Q_2=0 and Q_0=0 in relation (3.15), it becomes (3.17).

$$A = -Q_1 \quad , \quad B = -Q_1 \quad , \quad C = 0 \quad , \quad D = -\sqrt{3}Q_1 \tag{3.17}$$

Replacing (3.17) in (3.16), the intensity of the electric field by the positive sequence charge is derived in (3.18)

$$E=\frac{Q_1 s}{2\pi\varepsilon_0 RR'}\cdot\left[\frac{\left(R^2+R'^2-2RR'\cos(\theta_R+\theta_{R'})\right)}{\left(R'^4+s^4-2R'^2s^2\cos2\theta_{R'}\right)\left(R^4+s^4-2R^2s^2\cos2\theta_R\right)}\right]^{1/2}\cdot$$
$$[s^2\cdot\left(\left(R^2\cos2\theta_R+R'^2\cos2\theta_{R'}-s^2+RR'\cos(\theta_R-\theta_{R'})\right)^2+\left(R^2\sin2\theta_R-R'^2\sin2\theta_{R'}+RR'\sin(\theta_R-\theta_{R'})\right)^2\right)$$
$$+3\cdot\left(\left(R^2R'\cos(2\theta_R-\theta_{R'})+RR'^2\cos(\theta_R-2\theta_{R'})\right)^2+\left(R^2R'\sin(2\theta_R-\theta_{R'})+RR'^2\sin(\theta_R-2\theta_{R'})\right)^2\right)]^{1/2}$$

(3.18)

Relation (3.18) is also quite extensive. Taking account of the considerable deviation between the approximate values and the accurate values of the electric field, it results that the approximate formula (3.18) is not efficient.

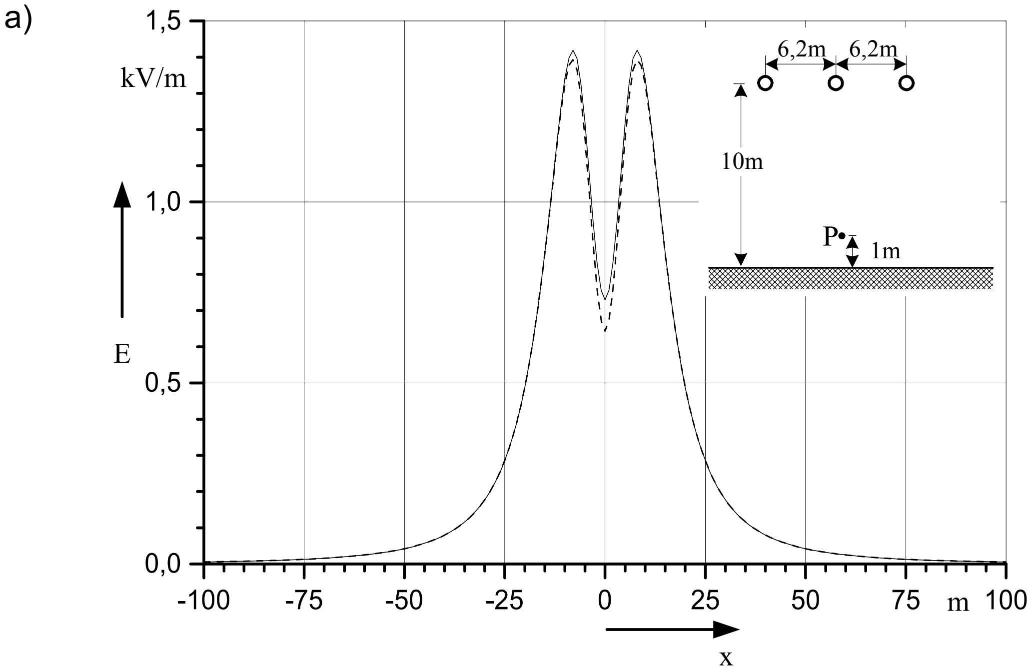
Another formula may be derived considering R=R´ and $\theta_R=\theta_{R'}$. Thus, replacing R=R´ and $\theta_R=\theta_{R'}$ in (3.16), the simpler formula (3.19) arises.

$$E=\frac{s}{\pi\varepsilon_0 R}\cdot\frac{\left|\sin\theta_R\right|}{\left(R^4+s^4-2R^2s^2\cos2\theta_R\right)}\cdot\left[\left(2AsR^2\cos2\theta_R-As^3+BsR^2+\frac{C}{s}R^4\right)^2+\left(2DR^3\cos\theta_R\right)^2\right]^{1/2}$$

(3.19)

Relation (3.19) is the accurate formula of the intensity of the electric field on the surface of ground. This relation is also a very good approximate formula for the electric field, for low calculation heights (up to 1m), as well as for far distances from the line.

Fig. 5 shows the comparison of the electric field E produced by a single circuit line with the conductors in horizontal arrangement, as it is calculated by the accurate formula (3.16) and the approximate formula (3.19). In fig. 5a the comparison between the two formulas is made for calculation height 1m from the ground, while in fig. 5b the comparison is made for calculation height 3m from it.



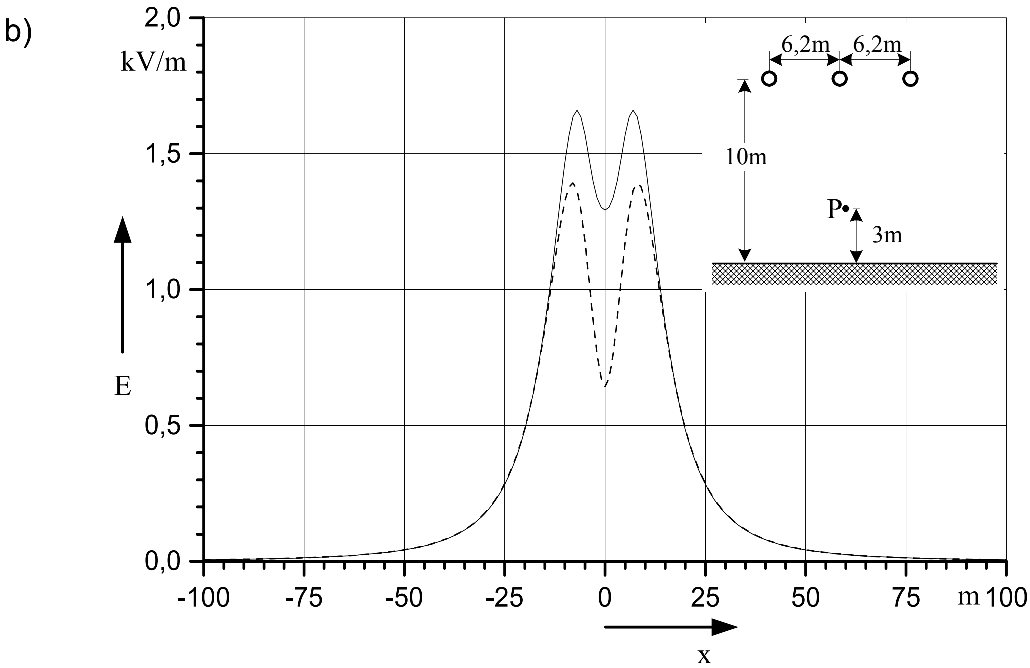


Fig. 5. Intensity of the electric field E produced by a typical 150kV single circuit line with horizontal arrangement of the conductors, as it is calculated from the accurate formula (3.16) and the approximate formula (3.19) for height a) 1m and b) 3m from the ground.
__________ accurate formula (3.16) --------------- approximate formula (3.19)

Fig.5a shows that for low calculation heights (up to 1m), the approximate formula (3.19) gives the intensity of the electric field with exceptional precision, not only far from the line, but also close to it. Fig 5b shows that for bigger calculation heights the formula (3.19) gives satisfying results only for far distances from the line, while close to it the deviation is significant.

4 Conclusions

The electric field produced by power lines of big length in comparison with the distances between the conductors and the distances between the conductors and the ground, can be expressed with extensive analytical formulas. In the present paper a method to derive the analytical formulas of the electric field has been developed. This method is based on the analysis of the intensity of the electric field into two multipole expansions, one from the real conductors system and the other from the images conductors system. For the simplification of the mathematical expressions double complex numbers are used. Due to the asymmetry of conductors charges, they are analyzed in their symmetrical components. Using this method the accurate analytical formula for the intensity of the electric field produced by single circuit lines with the conductors in horizontal arrangement is given. This relation may be used for the accurate calculation of the intensity of the electric field, at any point in the vicinity of the lines, close to or far from them. From the accurate formula, a simpler approximate formula with known precision is derived, which is valid either for low calculation heights (up to 1m) either for bigger heights in far distances from the line.

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